I.V. Denisov, Yu.N. Kulchin, A.V. Panov, N.A. Rybalchenko. — Optical Memory & Neural Networks, 2005, **14** (1), pp. 45–58

Neural network methods of reconstruction tomography problem solutions

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ABSTRACT

We review the papers devoted to solution of reconstruction tomographic problems by using neural networks. Recent developments in the solution of linear and non-linear tomographic problems in various types of tomography are surveyed.

Keywords: tomography, tomographic problem, neural network, perceptron, sensor, fiber optic, electrical impedance tomography, image reconstruction

INTRODUCTION

In the past few decades the problem of the tomographic reconstruction of images and physical fields (objects from hereon) distribution functions has become vital due to fast evolution of computers which allows creation and application of methods to processing information from measuring devices [1].

Existing measuring systems vary in the kinds of information carriers and components depending on type and size fed of objects under study. The methods of information gathering by the measuring systems and of subsequent processing of this information depend on the imposed requirements to the reconstructing tomography approaches.

Basically, both parallel and serial data gathering techniques may be implemented. The first technique enables us to gather large data arrays within one step. But this method has a very complex implementation architecture. The evident advantage of the second approach is its implementation architecture sim-

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plicity while its data processing rate that has a significant effect on the reconstruction of the greatly extended objects is lower.

The choice of pure computer or preliminary hardware solution of mathematical algorithms used determine the distinction of devices for processing gathered information. In the first approach the mathematical algorithms are implemented as software. Therefore, the devices gathering information from detecting systems make its elementary transformations into digital form. In the second case, mathematical algorithms are realized in hardware carrying out the required functional transformation of the obtained information. Then, the processed data is transferred to the computer for interpretation and visualization.

Mathematically, the tomographic problem statement consists in the reconstruction of the studied object parameters using integral data obtained from measuring lines. Generally, in the tomographic problem the number of equations is much fewer than the number of elements of images or fields studied. As a result, the number of the unknowns in the equations arising in tomographic problem solving exceeds the number of the equations. In this connection the problem of reconstruction of the studied parameter using incomplete integral data is ill-posed [2].

One can discern two groups of tomographic problems among existing types with the increasing complexity of solution. This complexity is connected with geometrical and physical characteristics of the object studied and with investigation method [3]:

1. Linear problems of computerized tomography arising in the case of usage of rectilinear measuring lines, whereas the integral signal is a superposition of signals from segments of the measuring line;
2. Nonlinear tomographic problems arising when curvilinear measuring lines or non-linear dependence of integral data on the measured value caused by physical properties of signal carrier are used.

At present a lot of algorithms applied to the reconstruction of the information about distributed physical field parameters have been developed. This is related to both the extensive variety of tomographic problem definitions and to ways of their experimental realization and to persistent efforts of researchers to develop such an algorithm which would surpass the exiting ones at least in one of the following characteristics: operating rate, restrictions on the memory, resolution, contrast, number of required projections etc. [3–5]. The reconstructing algorithms used for solving linear tomographic problems can be separated into three groups of methods [4]:

— algebraic (methods of regularization);
— integral (methods of filtration);
— analytical (methods of approximation).

In solving the nonlinear tomographic problems one has to adapt the existing algorithms to the experiment conditions which, as a rule, leads to growing requirements to computational capacity and to simultaneous reduction of reconstruction accuracy. In reality the measuring line paths, the magnitude and nature of the investigated object parameter on the integral signal magnitude can vary which also lowers the accuracy of reconstruction by the afore-mentioned methods.

Neural network (NN) utilization is an alternative approach to tomographic problem solution. Neural networks have a number of favorable features, the most important qualities of them being adaptability and generalization. The adaptability of NN to specific conditions of the problem is the result of training. The generalization of a trained NN lies in its ability to solve a problem for initially unknown for NN distributions belonging to the same class as the ones used for learning.

Among the most frequent types of NN applied to solving the tomographic problems one can specify a perceptron and an NN with feedback (Hopfield NN).

The perceptron is a feedforward NN consisting of several layers of neurons. The signal transfer in this NN is performed only in one direction: from the input layer to the output one, with the neurons of one layer being coupled only with the neurons from another layer [6]. Perceptrons may have only one layer of neurons with a simple and transparent architecture and limited capabilities. As a rule, such NN's are used for particular classification of smooth functions. Since this linear separation restricts the perceptron representation capabilities [7] additional layers are used forming multilayer perceptrons applied to general classification by convex surfaces [8].

Fig. 1 shows the architecture of a three-layer perceptron which is most frequently applied to solving the tomographic problems. The first layer of neurons serves as NN inputs forming the input vector $\mathbf{X} = \{x_1, x_2, \ldots x_K\}$, where $x_1, x_2, \ldots x_K$ are integral data detected by the measuring system, K is the number of neurons in the input layer being equal to the number of measuring lines. Besides this, the first layer has no other function. The second layer of NN (the so called hidden layer) takes the transformation of the form:

$$s_j = f\left(\sum_{k=1}^{K} \bar{w}_{jk} x_k\right) + \bar{\theta}_j,\ j = 1,2\ldots J \tag{1}$$

where s_j are output states of the second layer neurons, $\bar{w}_{jk}$ are elements of the matrix of inter-neuron coupling between the first and second layers, determining the coupling between kth neuron of the first layer and jth neuron of the second layer, $\bar{\theta}_j$ are threshold potentials of the hidden layer, J is the number of neurons of the second layer, f is empirically selected activation function, which is usually linear, sigmoid (e.g. unipolar sigmoid or hyperbolic tangent), radial-basis dependencies [9]. The output layer of the neurons typically makes the linear transformation:

$$y_i = \sum_{j=1}^{J} w_{ij} s_j + \theta_i,\ i = 1,2\ldots L, \tag{2}$$

where y_i are activations of the output layer of the neurons, w_{ij} are elements of the matrix of inter-neuron coupling between the second and the third layers, θ_i are threshold potentials of the output layer, L is the number of the third layer neurons. The number of the neurons in each layer is usually selected in accordance with the conditions of the problem solved.

The process of selection of the elements of the coupling matrix and vectors of threshold potentials $\bar{w}_{jk}$, w_{ij}, $\bar{\theta}_j$, θ_i under minimization of the deviation between the required and computed output values intended for solution of some problem is referred to training of NN. Training of the perceptron is carried out with training patterns being a set of pairs of NN input vectors **X** and corresponding initially known output vectors $\mathbf{Y} = \{y_1, y_2, \ldots y_L\}$. Perceptron training is an optimization problem which is, as a rule, solved by such methods as: gradient, simulated annealing [10], their combinations etc. It was shown in papers [11,12] based on Kolmogorov theorem [13] that the perceptron with nonlinear monotonic finite function of activation is capable of approximating continuous mappings to any desired degree of accuracy, so the perceptron can solve the problems of reconstruction tomography.

NN's with feedback, for first time developed by Hopfield for stepwise activation function, have a peculiarity of dynamic NN response, i.e. the computed output cyclically modifies the input till achievement of the required accuracy of the information restoration [14]. Such NN's are divided into stable or unstable ones using the attribute of stability or inconstancy of the output, accordingly. A subset of feedback NN's in which outputs finally reach a stable state is described by the sufficient condition of stability for the first time derived in Ref. [15]: if the matrix of weight coefficients is symmetric and its leading diagonal consists of zeros then this NN always converges to a stable point. However there may exist stable NN with non-symmetric matrix of weight coefficients and non-zero diagonal elements as well as NN in which small deviations from sufficient condition lead to loss of stability.

In Hopfield NN the zero layer **X** does not take a computational function but only feeds the outputs of NN backward to the inputs (Fig. 2). Each of the neurons of the first layer **Y** calculates a weighted sum of its own inputs, giving the signal which is then transformed by NN into the output signal *OUT*. At this, the solution of the reconstruction problem falls into the following steps:

1. An energy function is constructed so that a global minimum of this function coincide with the problem solution.

2. The feedback is broken and the input vector *IN* feeds into the NN. The output values are calculated.

3. The feedback is closed and the NN is enabled to independently change its state (the relaxation). The relaxation process stops after the output vector becomes constant, i.e. the energy function minimum is reached. The NN outputs obtained solve the problem.

The NN with feedback perform the function of associative memory. It means that the NN using the vector fed on the input creates on the output one of the vectors remembered before which is most similar to the given input vector, in a certain sense.

The Hopfield NN can work as autoassociative or heteroassociative memory. In the first mode the vector most resembling the input vector is called from the library. The second mode of the heteroassociative memory requires training with a teacher using the sigmoid transfer function. In this case the NN works continuously and reliably reaches the global minimum of the deviation error [16].

In the Hopfield NN the memory matrix is formed by mutual external multiplications of the library vectors with subsequent summation. The library vector is called by the vector-matrix multiplication of the input data and the memory matrix. The derived vector is further exposed to functional transformation and used as the input data for the next iteration [17]. This iterative process repeats until the required convergence is achieved.

The drawback of the Hopfield NN's is their tendency to stabilize at the local minimum instead of global one. This difficulty is usually overcome by means of the NN class known as Boltzmann machines, in which the neuron state changes obey statistical laws instead of deterministic ones [18].

The method of data retrieval used in Hopfield NN is called addressing by contents. It is widely used in biological NN's and is highly promising for the creation of systems for the recognition of signals and images.

In this paper we present a review of the modern NN methods for solving both linear and nonlinear problems of the reconstruction tomography.

APPLICATION OF NN'S TO THE RECONSTRUCTION OF IMAGES FROM PROJECTIONS

The procedure of the reconstruction of images from the projections which is a linear tomographic problem lies in finding of an unknown vector $\mathbf{X} = \{x_j\}$, containing information about each pixel of the image or about the investigated field parameter in a certain point, using known components of the vector of the integral measurements $\mathbf{Y} = \{y_i\}$, which is a sum:

$$\mathbf{Y} = \mathbf{AX} + \mathbf{N}, \tag{3}$$

where $\mathbf{A} = \{a_{ij}\}$ is the known projection matrix, **N** is the vector of random errors. These problems are usually solved by the well known classical methods, e.g. algebraic reconstruction technique, filtered back projection, series expansion etc. [4]. However, with large number of pixels and image density levels these methods consume high computational capacities. Besides, the use of iterative methods does not always restore the images with sufficient quality while the work takes much time. At the same time, the NN use for tomographic problem solving is particularly promising in function reconstruction from small amount of incoming data [19]. Therefore, for this class of problems [20–24] the use of NN's was proposed. Most works in this area are based on the application of Hopfield NN [20,21].

Let us briefly describe the modified Hopfield NN utilized for solving the problems of computerized tomography. In the tomographic problem solving approach based on series expansion assuming negligibility of noise **N,** the solution of the tomographic problem can be reduced to the optimization of the norm ("energy"):

$$g(x) = \frac{1}{2}\sum_{j=1}^{n} x_j^2 \tag{4}$$

under the constraints imposed on **X**: $x_{j\min} \le x_j \le x_{j\max}$, $j = 1, 2 \ldots J$, where x_j are the components of vector **X**. Instead of the norm (4) one can use enthropy function, for example, the Shannon enthropy taken with the minus sign:

$$g(x) = \sum_{j=1}^{n} x_j \ln x_j, \quad x_j > 0 . \tag{5}$$

The optimization problem can be solved by means of Hopfield NN. For this purpose a general energy function is introduced:

$$E(\mathbf{X}) = \nu g(\mathbf{X}) + \sum_{i=1}^{m} p\left(r_i(\mathbf{X})\right), \tag{6}$$

where the penalty parameter ν is selected as a positive number, $r_i(\mathbf{X}) = \sum_{j=1}^{n} a_{ij} x_j - y_j$. The penalty function p can take various forms, e.g. square dependence [21]:

$$p(r) = \frac{1}{2} r^2 .$$

The minimization of the energy $E(\mathbf{X})$ reduces to solving a set of nonlinear differential equations which describes a modified Hopfield NN:

$$\frac{dx_j}{dt} = -\mu \frac{\partial E(\mathbf{X})}{\partial x_j} = -\mu \left[\nu \phi(x_j) + \sum_{i=1}^{m} a_{ij} \Psi(r_i) \right], \tag{7}$$

where the training rate μ is guessed as some positive number, $\phi = \dfrac{\partial g(\mathbf{X})}{\partial x_j}$ are activation functions of the output neurons, $\Psi(r_i) = \dfrac{\partial p(r_i)}{\partial r_i}$ are activation functions of the input neurons, t is time.

The calculations of Chichocki et al. [21] show that NN’s of this type can reconstruct such model images as, e.g. Shepp-Logan phantom [22] modeling a section of a human head and frequently used for testing the reconstruction tomography methods. Fig. 3 depicts the results of the reconstruction of a model image obtained by means of a modified Hopfield NN [21].

Afterwards, Wang and Wahl [23] proposed an improvement of the above approach on the basis of vector-enthropy optimization. They advanced the idea of using a linear combination of image enthropy, a function of standard deviation between original and reconstructed data, as an objective function g. This problem is also solved by the authors by means of a modified Hopfield NN. Comparison of reconstructed patterns made in Ref. [23] with the results obtained by the convolution method and algebraic reconstruction technique shows the advantage of the proposed NN approach.

The linear problems of reconstruction tomography can also be solved by means of perceptron NN's. For example Ali et al. [24,25] utilized the perceptron with nonlinear hidden layer the activation function of which was chosen in the form of a sigmoid. The training was done on computer-modeled measurements by means of error back propagation and simulated annealing. The authors demonstrated that the result of the NN model test data reconstruction is more accurate than the one obtained by algebraic methods.

APPLICATION OF NN'S TO RECONSTRUCTION OF RADIO-FREQUENCY ELECTROMAGNETIC TOMOGRAPHY DATA

The radio-frequency electromagnetic tomography is a method of acquiring information about the distribution of electromagnetic parameters (electrical impedance, conductivity or capacitance, magnetic induction) inside a certain object. The electrical impedance tomography (EIT) is most frequently used, for example, in medicine, for the study of complex liquid/gas pipe flows, in geophysics, etc. This type of tomography utilizes a system of electrodes mounted around an investigated object. This system enables us to measure the resistance between two electrodes pairwise. The reconstruction of the studied parameter distribution is a complex nonlinear problem which is solved with modified methods of solving of linear problems of computerized tomography [26,27] or, otherwise, with the development of specific methods [28]. As far back as early 1990s the application of NN's to solving this problem was advanced. Initially a simple NN of

ADALINE type was used but the reconstruction was rather crude [29]. Later, Nooralahiyan and Hoyle applied NN of perceptron type with one hidden layer to the reconstruction and simultaneous classification of three-component flows inside circular perimeter pipes [30]. The authors chose double-sigmoid as an activation function of the hidden layer. This method of the reconstruction of the flow fraction distribution and classification provided for simplifying the NN structure.

Afterwards, this approach was applied to the reconstruction of three-component flows studied with an experimental setup [31]. Examples of original and reconstructed flow sections are showed in Fig. 4.

The authors of Refs. [32,33] used perceptron in the algorithms of the simulated electrical impedance image reconstruction. These NN's have linear [32] or nonlinear [33] hidden layers with few inputs and outputs for the reconstruction of image segments. The use of this approach allowed the authors to significantly reduce requirements to computational power but this lowered the generalization capabilities of NN's.

A more complex problem of the electrical impedance distribution reconstruction obtained in medical surveys of patients was solved by Korjenevsky [34]. For this purpose he also applied two types of NN's: the two-layer linear NN and the perceptron described by equations (1), (2), in which of the neurons number of second layer was 60 and the number of neurons in the first layer was chosen to be equal to the quantity of input measurements; hyperbolic tangent was used as the activation function.

The sets of training patterns of about 1.5-2 thousand known distributions were used for training. The noise signal was added into learning patterns, the value of it being on the order of 1% of maximal input signal level. The author was utilized the error back propagation using conjugate gradient optimization.

Korjenevsky [34] also applied the proposed NN technique to solving of an inverse problem arising in magnetic induction tomography which uses the measurement of the magnetic field perturbations caused by eddy currents and allows finding the distribution of electrical conductivity in a specimen. This problem is similar to the inverse problem occurring in EIT.

The results of the reconstruction of randomly generated model distributions by the NN with nonlinear hidden layer are quite satisfactory (Fig. 5). The linear NN reconstructs the test distribution with lower accuracy than in the case of nonlinear NN or back projection technique with synthesis of reference data. The attempts to reconstruct experimentally obtained distributions of electrical conductivity by nonlinear NN trained by means of randomly generated model distributions did not yield any satisfactory result. The author also pointed of the deficiency of NN use — the necessity to create large amount of training patterns.

Besides the algorithms based on gradient optimization training the Bayesian approach can be applied [35]. This method allows more effective training of NN solving the inverse problem of EIT but it requires prior information about the distribution being reconstructed. Vehtari and Lampinen showed [36] that a NN trained with a Bayesian algorithm more accurately than with gradient optimization one, reconstructs the modeled distributions representing gas bubble formation inside a pipe with liquid (two-component medium).

Warsito and Fan applied the technique worked out on the basis of modified Hopfield network [23] to the image reconstruction for flow systems of media consisting of various states: gaseous, liquid, solid [37,38]. The authors selected the superposition of negative image enthropy, weighted square error function between measured and estimated data and sum of non-uniformity and peakedness functions as an objective function for the optimization. Warsito and Fan reconstructed real data obtained with an experimental setup. They showed that the test flow sections were reconstructed more accurately by the NN than by means of a linear back projection or simultaneous image reconstruction technique [39].

APPLICATION OF NN'S TO DATA RECONSTRUCTION OF FIBER-OPTIC TOMOGRAPHY

The fiber-optic tomography is the method of the information reconstruction of physical field parameters (temperature, deformation, impact detection, etc.) of data obtained from fiber-optic measuring line arrays. Such lines are stacked under the required scanning scheme on the investigated areas and form a fiber-optic sensor array [40]. The fiber-optic sensor array, light beam detectors and devices processing obtained information form fiber-optic sensor system. The optical signals on the output of measuring lines form data array, which contains the information on parameters of physical fields. Real-time processing mutable optical signals from all fiber-optic measuring lines of the system is extremely important. Fiber-optic measuring systems have variety of exclusive advantages. It is connected to widely known features of fiber-optic element base in comparison with devices on the basis of other elements: wide bandwidth of optical fiber, its insensitivity to electromagnetic noise, small weight, complexity of realization of the illegal access to optical information and other characteristics of fiber [41,42].

Refs. [43–45] show results of studied distribution reconstruction by means of linear perceptrons. Training of the NN and reconstruction of test images was made for smooth distributions. The authors used the modification delta-rule of elements of the matrix of connections w_{ij} for training:

$$\Delta w_{ij} = \varepsilon x_i (y_j - \tilde{y}_j), \tag{8}$$

where ε is the parameter specifying training rate, x_i is the state of ith neuron of the input layer, $\tilde{y}_j$ are values of outputs of NN for the training pattern. Kulchin et al. [44,46] found expression for optimal value of ε for linear perceptrons. Fig. 6 depicts results of reconstruction of the simulated test image of the array of 8×8 measuring lines [43]. For this purpose the two-layer perceptron NN with 31 input and 64 output neurons was modeled. The NN formed the matrix of connections for fiber-optic measuring system being capable to reconstruct the distribution of investigated physical field. Formation of the matrix of connections has demanded 22500 cycles at 32 training pairs.

Afterwards, Kulchin et al. [45] using above-mentioned simulation represent NN implemented as a set of amplitude holograms recorded on disk holographic carrier. The NN was intended for processing output data from distributed fiber-optic measuring system. It is experimentally shown, that this system allows to reconstruct functions of spatial distribution of the studied physical parameter in a certain point with the error not exceeding 20%.

Later, the model of three-layer perceptron with one nonlinear hidden layer was used for solving the tomographic problem [47–49]. Transition to such type of NN allowed to restore more complex distributions of the investigated field, e.g. peaks in a form of Gaussian distribution being more close to practice. So, Kulchin et al. [50] made the mock-up of the fiber-optical sensor system detecting the field of acoustic fluctuations. A sketch of this mock-up with the array size 4×4 is shown in Fig. 7.

The NN, used in Refs. [47–49], reconstructing the data obtained by the mock-up of the interferometric fiber-optic sensor system [50], was described by the equations (1), (2); with hyperbolic tangent being an activation function. For the sensor array shown in Fig. 7, the input and hidden layers contained $4n-1$ neurons being equal to number of fiber-optic measuring lines, and the third output layer contained $n\times n$ neurons, corresponding the number of sites of the measuring system. The NN could not have threshold potentials, in that case it was considered that $\overline{\theta}_j \equiv 0$ and $\theta_i \equiv 0$.

The reconstruction accuracy of the investigated physical field distribution was defined by quality of NN training. The deviation between original and restored by NN distributions was defined by expression:

$$D = \frac{1}{2}\sum_{\mu,i}\left(y_i^\mu - \tilde{y}_i^\mu\right)^2, \tag{9}$$

where μ is the number of pair of input and output vectors from training page, $\tilde{y}_i^\mu$ is the required state of the output neuron. The combination of gradient methods and elements of simulated annealing made available effective NN training. The training process stopped after a certain count of objective function D iterations being on order of several millions. Fig. 8 shows results of NN reconstruction of the acoustic oscillations field studied by the mock-up of fiber-optic sensor system [48].

Kamenev et al. [49] added noise into learning patterns in order to more effectively train NN. This noise simulated random errors arising from measurements, the limited accuracy of sensors, etc. For this purpose the set of training patterns was increased in three times due to addition of the vectors formed as follows:

$$\tilde{y}_{i,\text{noise}}^{\mu} = \tilde{y}_i^{\mu}\left(1+\eta\right), \tag{10}$$

where $\tilde{y}_{i,\text{noise}}^{\mu}$ is the training pattern with noise addition, η is the random number from the interval $\left[-\varepsilon,\varepsilon\right]$, ε is "noise intensity". The authors in detail investigated influence of "noise intensity" on rate and quality of perceptron training, and also on accuracy of reconstruction of test distributions.

Existence of both positive and negative aspects of NN's encourages researchers to combine various NN's with as well other mathematical algorithms as among themselves to obtain new highly effective computing algorithms. So, Kulchin et al. [51] presented promising algorithm on the basis of the combination of algebraic methods carrying out preliminary processing of information which is then fed onto perceptron NN. This NN had 31 neuron in the input and 64 neuron in the output layers.

Tu and Huang [52] advanced two new combined NN methods of data processing defining coordinates and values of external deformation. This methods combine perceptron with one hidden layer and Kohonen NN [53]. The authors using numerical experiment showed that application of Kohonen NN to solving the problem of localization of impact position on fiber-optic sensing array of lines of size 4 × 4 in the combination with nonlinear perceptron allows to reach the error of mismatch 10^{-6} in estimation of magnitude of external impact using less than 8000 training cycles. The result obtained by the authors allowed to use Kohonen NN for localization of position of impacts in fiber-optic tomographic problem.

OTHER TYPES OF TOMOGRAPHIC PROBLEMS

The studies of NN application to the restoration of tomographic data are not restricted to the afore mentioned problems. One more area of NN application in tomography is the reconstruction of data obtained with the help of single photon emission computerized tomography (SPECT) and positron emission tomography (PET). Both types are frequently utilized in medicine and are based on the detection of collimated γ-radiation. In the case of PET, two photons are detected which are emitted in opposite directions resulting from annihilation of positrons arising from β^+-decay of the isotopes introduced into the patient blood. For single photon emission computerized tomography the isotopes with β^--decay and simultaneous emission of γ-quanta are utilized. PET uses more short-living isotopes which provide for higher space and time resolution but, because of the fast decay, these isotopes must be prepared by means of cyclotron just before the survey. With the use of these methods a linear tomographic problem arises which is usually solved by the filtered back projection technique. But this algorithm may not give acceptable reconstruction quality, at that, this technique may not reconstruct the image in real-time.

Comtat and Morel [54] applied a self-organizing Kohonen NN to the reconstruction of data obtained from the PET simulation. Although the data reconstruction accuracy was lower than in the case of filtered back projection technique the authors indicate the independence of the used NN approach on the number of utilized sensors which is important for universality of the computational algorithms.

The utilization of the perceptron appeared to be more productive for solving this tomographic problem [55–58]. Kerr and Bartlet [52] used for reconstruction of SPECT data the perceptron with nonlinear layer implemented on the basis of a massively parallel SIMD computer (SIMD means single instruction, multiple data). This computer consists of a large number of simple processors interconnected into an array (Kerr and Bartlet used the array of 64×64 processors) controlled by a more powerful processor. This type of computer has the property of working in parallel. Each processor of this SIMD computer performed the role of a separate neuron. The use of this parallel system allowed the authors to attain fast training of NN. The authors showed that the trained NN is capable of rather correct visualizing test images of organs. Moreover, Kerr and Bartlet indicated that the NN trained on SIMD computer can subsequently be used on an ordinary computer. Bevilacqua et al. [59] used the same approach to training by means of a SIMD computer, but with the utilization of the MADALINE NN, a two-layer linear perceptron, for the restoration of the PET data.

Rodriguez et al. [56] showed that the NN, trained with test patterns in the form of Gaussian distributions is capable of reconstructing images of phantoms of a human head section and of letters.

We should also note the studies on the application of NN's to the reconstruction of ultrasonic tomography data. Hutchins et al. [60] applied a multilayer perceptron to the restoration of simulated and experimental data of a 4×4 pixel array. NN proved to be able to locate the presence of defect in the area being investigated; however, the obtained localization was very approximate, that might be due to the limited computational capacities insufficient for training an NN.

CONCLUSIONS

As follows from the above, NN's can be used for solving the inverse problems of tomography. As a rule, for solving these types of problems it is effective to use NN's of two types: the Hopfield network and the perceptron. The merits of the perceptron use for the solution of reconstruction tomography problems include high rate of data processing, ability of generalization, the simple network structure, which can be produced on the basis of both electronic and optical elements. The deficiencies of these NN's are the necessity of using a large quantity of training patterns and the high duration of the training process. It should be noted that if the simultaneous classification of processed data is required, then this task can also be solved by means of a perceptron. However, the methods selection of the training patterns and combining this NN with other computational algorithms call for further study.

The Hopfield networks are usually used for the solution of linear tomographic problems. The merit of this class of NN's is the absence of the need of using the large number of training patterns, the deficiencies are: limited possibilities of generalization and difficulty of realization in the form of an optoelectronic NN.

We should acknowledge the advantages of optical methods of NN's implementation solving the tomographic problem, in particular, with the aid of the holograms [45], based on the results of works [61,62], and also the realization of perceptron in the form of collected optical neurochips, built on the basis of planar waveguides and prisms [63], which permit parallel processing of integral data.

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FIGURE CAPTIONS

Fig. 1. Schematic diagram of a three-layer perceptron neural network.

Fig. 2. Hopfield neural network with two layers.

Fig. 3. Reconstructed by the NN [21] Shepp-Logan phantom image. (*a*) squared relationship $g(x)$ in formula (6), (*b*) $g(x)$ is negative Shannon entropy (5).

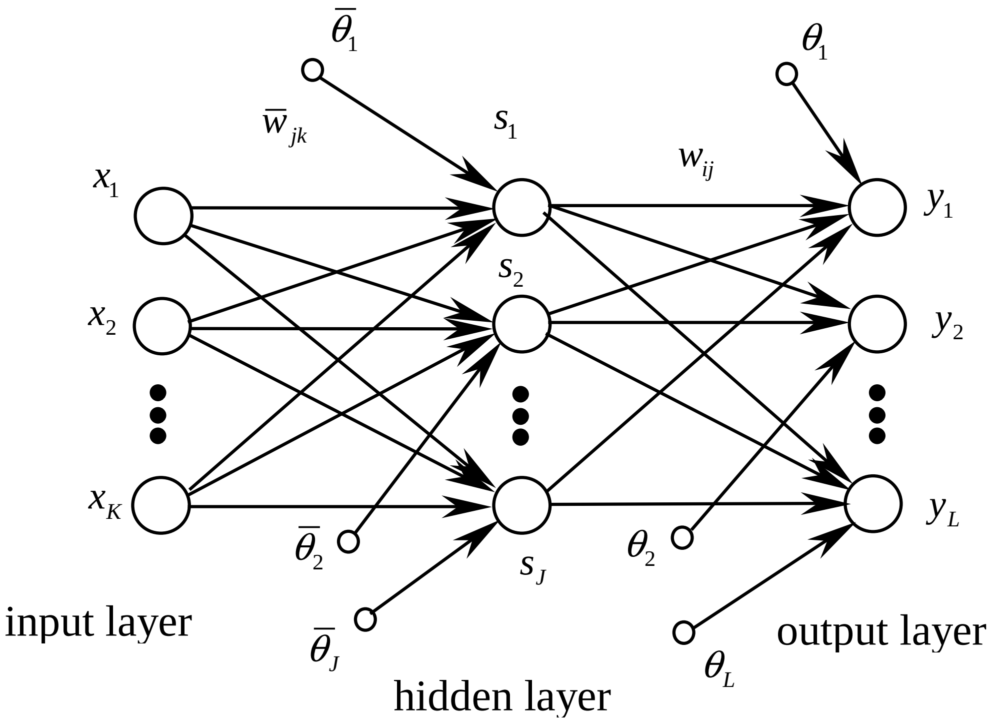
Fig. 4. Results of reconstruction of experimental data of electrical impedance tomography [31]. On the right of figure original data are shown, the left part depicts NN reconstruction. Different gradations correspond to various components of studied mixture.

Fig. 5. Results of EIT visualization of simulated electrical conductivity distributions by the perceptron with nonlinear hidden layer [34]. On the left part of the figure original data are displayed, on the right of the figure the results of NN reconstruction are depicted.

Fig. 6. The original distribution (a) and result of its reconstruction by two-layer perceptron with linear activation function (b) [43].

Fig. 7. The architecture of fiber-optic sensor system.

Fig. 8. The original (a) and reconstructed by the NN (b) acoustic field distribution [48]. The original distribution was detected with the mock-up of the interferometric fiber-optic measuring system with sizes 4×4 [50]. The dots on base plane depict isolines.



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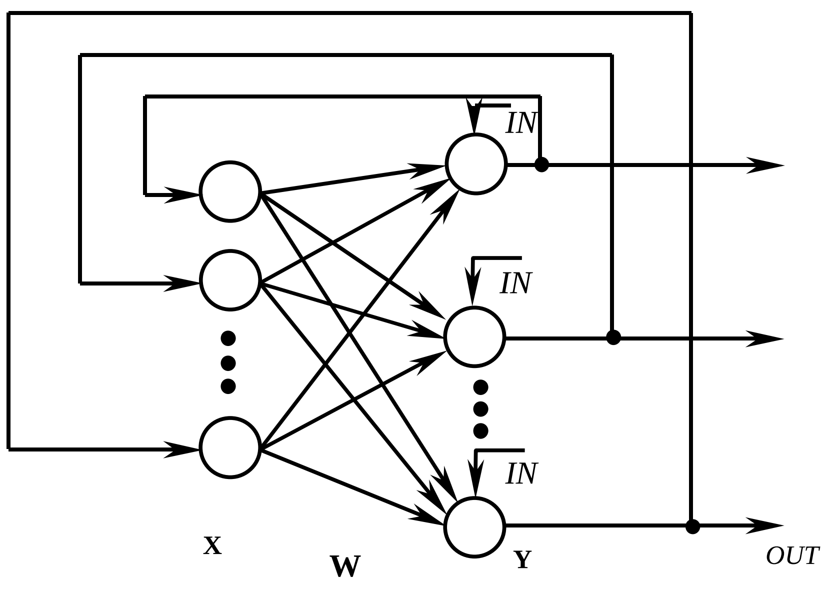

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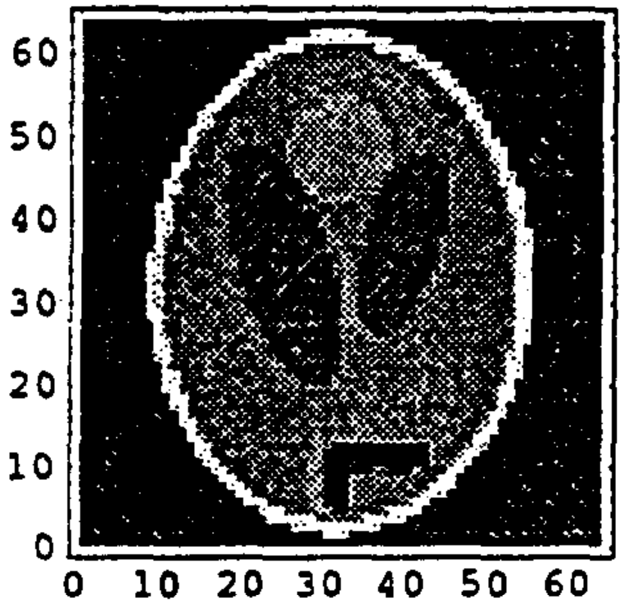
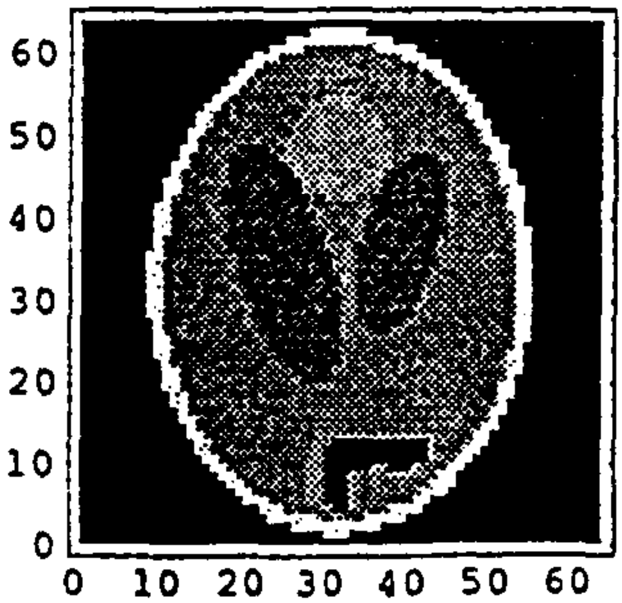


(a)



(b)

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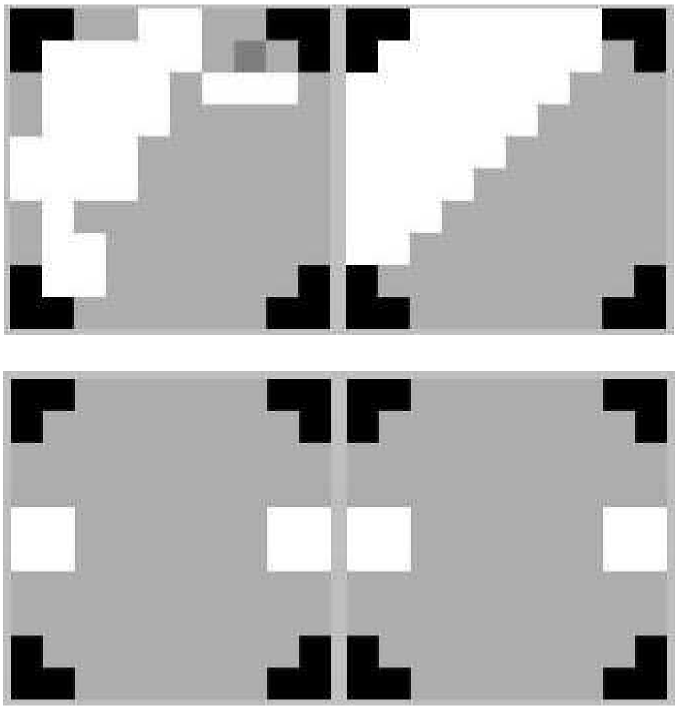

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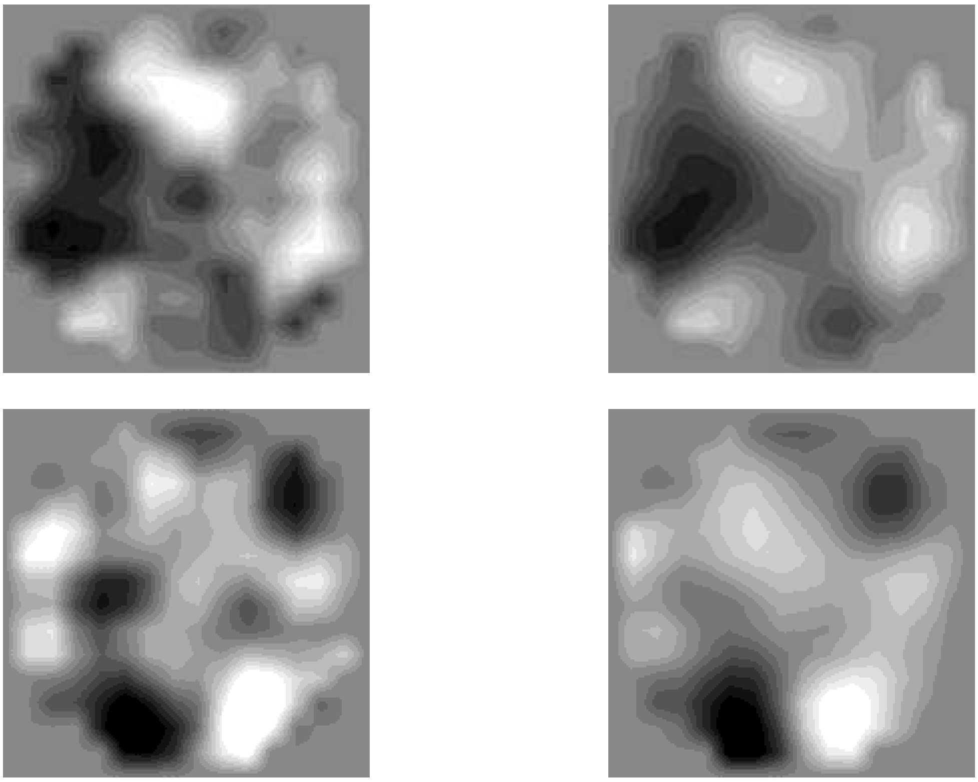

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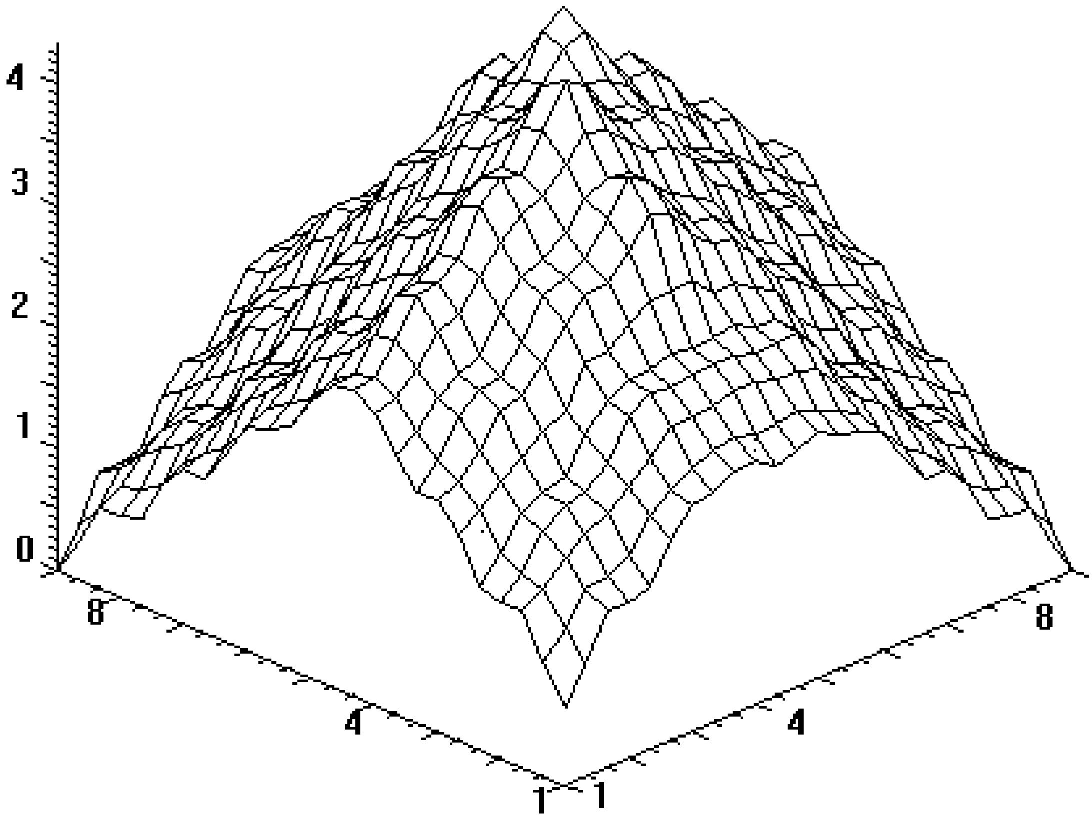
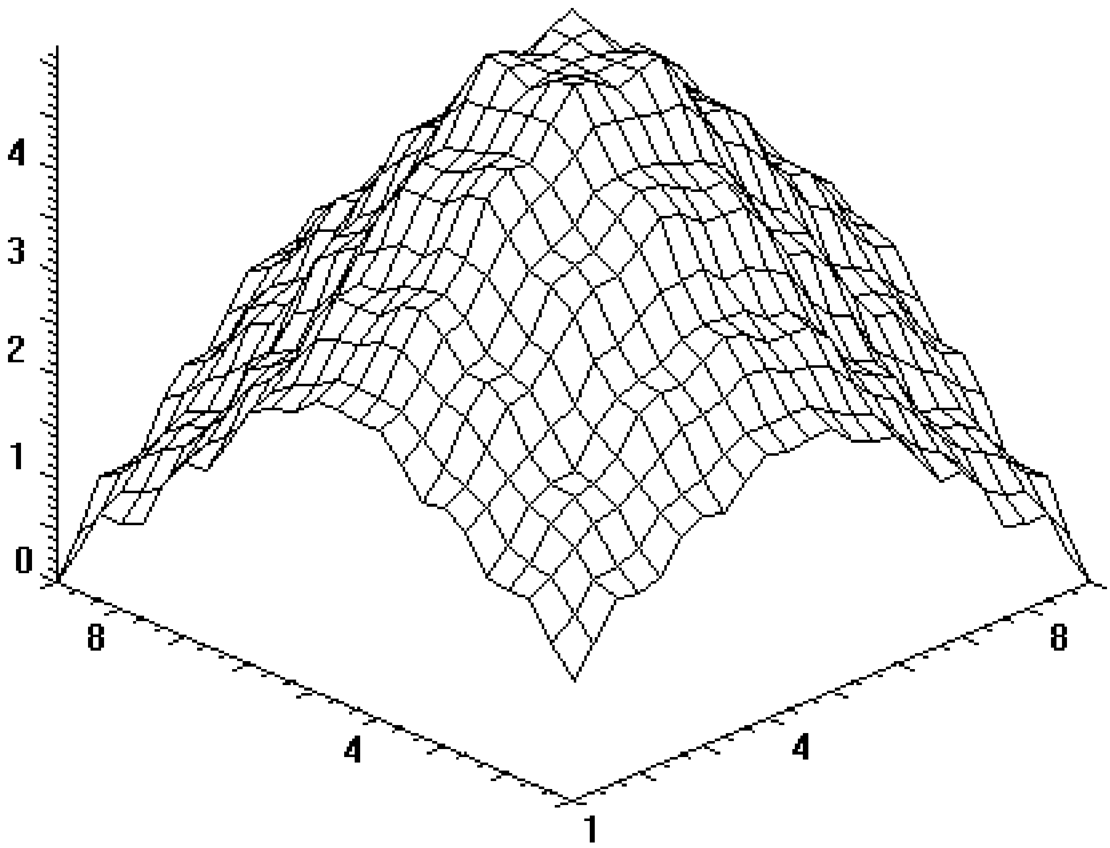


(a)

Fig. 6.



(b)

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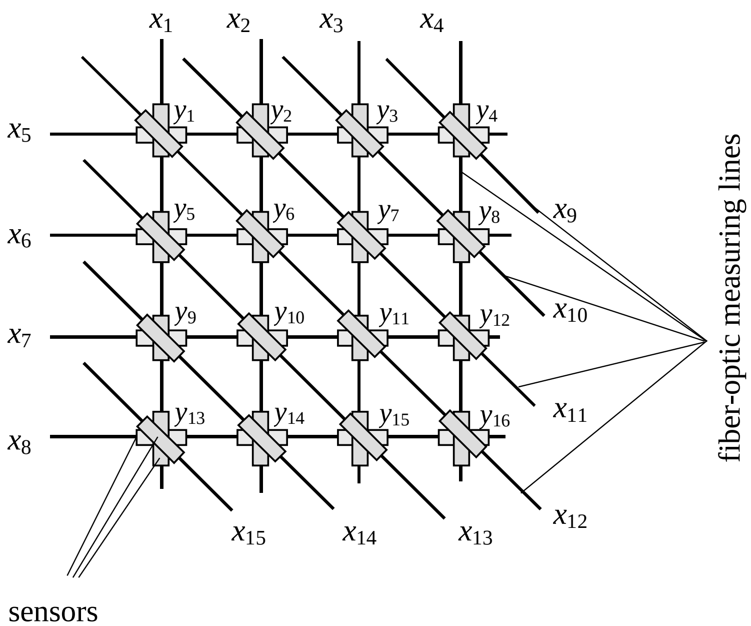

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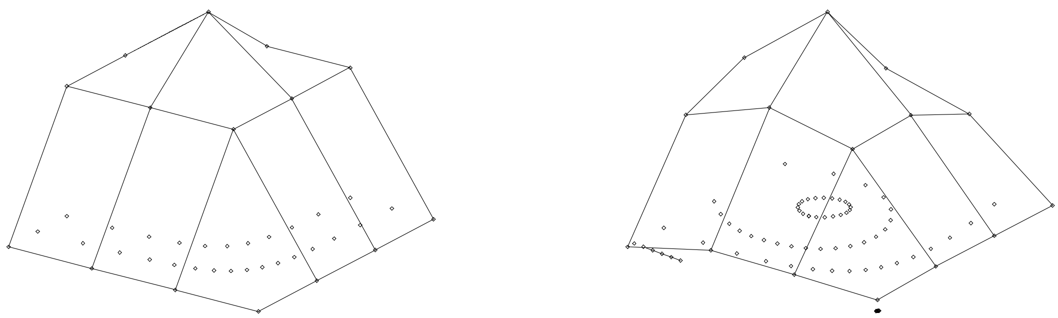

(a) (b)

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