ISSN 1069-3513, Izvestiya, Physics of the Solid Earth, 2009, Vol. 45, No. 1, pp. 63–69. © Pleiades Publishing, Ltd., 2009.
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Piezoremanent Magnetization of the Ensemble of Single-Domain Particles

L. L. Afremov[a] and A. V. Panov[b]

[a] *Far East State University, ul. Sukhanova 8, Vladivostok, 690600 Russia*
e-mail: afremovl@mail.dvgu.ru
[b] *Institute of Automatics and Control Processes, Far East Division, Russian Academy of Sciences*
e-mail: panov@iacp.dvo.ru

Received July 1, 2008

Abstract—Using the model of single-domain particles, a theoretical analysis of the properties of different forms of piezoremanent magnetization is carried out. The relationships which determine piezoremanent magnetization within the entire range of stresses are obtained. Two mechanisms of magnetization are revealed. It is shown that in the weak field approximation the magnetization of the first type has a quadratic dependence on the magnetic field and weakly depends on stresses, while another type of magnetization, which mainly determines the piezomagnetic effect, is proportional to the field and stresses. Moreover the magnetization that forms in the course of a change in stresses parallel to the field is always higher than the magnetization that appears as a result of action perpendicular to the field.

PACS numbers: 91.25.Ng

DOI: 10.1134/S1069351309010091

Seismic and volcanic processes that vary the distribution of internal mechanical stresses in the Earth's crust can lead to the piezomagnetic effect, which eventually exerts a change in the geomagnetic field. The study of the piezomagnetic effect makes it possible to investigate magma intrusions and volcanic upheavals, to carry out monitoring of accumulation of seismic stresses, and to determine the changes in stresses during earthquakes [Uyeshima, 2007]. The advantage of this method is the ability to directly evaluate a change in mechanical stresses under the surface, comparing the experimental data with the known solutions for particular types of sources of elastic stresses, for example, faults and point sources of expansion or dislocations.

Piezomagnetic research [Del Negro et al., 2004] has shown that in the case of volcanic eruptions, the change in the geomagnetic field reaches 10 nT, while in the case of seismic phenomena, the changes in the field are smaller by an order of magnitude [Johnston et al., 1994; Uyeshima, 2007]. Such a substantial difference stems from the deeper occurrence of sources of seismic stresses in comparison with volcanic phenomena.

An additional reason for the change in the intensity of geomagnetic field can be the accumulation of stresses as a result of subduction process [Nishida et al., 2004]. Modeling of this process [Nishida et al., 2004] has shown that the experimentally observed change in the geomagnetic field coincides with quantitative assessments if the sensitivity of magnetization to stress is higher by an order of magnitude than the sensitivity predicted in [Stacey and Johnston, 1972; Nagata, 1968, 1970], theoretical studies of which are based on the model of multidomain particles [Nagata and Carleton, 1969].

Therefore, our study of the properties of piezoremanent magnetization of the system of single-domain particles can expand the basis of piezomagnetic research.

If uniaxial mechanical stresses are applied (σ_+) or removed (σ_0) in the presence of magnetic field (H_+), then so-called piezoremanent magnetization $I_r(H_+\sigma_+\sigma_0H_0)$ appears [Ohnaka, 1969], which can significantly exceed the isothermal remanent magnetization $I_r(H_+H_0)$. It has been experimentally shown [Nagata, 1970; Nagata and Carleton, 1968, 1969] that within the range of weak fields and low mechanical stresses, magnetization $I_r(H_+\sigma_+\sigma_0H_0)$ is proportional to field H and stresses σ. Moreover, it depends on the mutual orientation of field and stresses. The transversal piezoremanent magnetization ($H \perp \sigma$) in spite of the increase with an increase in small σ is always by 10–25% less than the longitudinal magnetization is generated in the field parallel to the stresses ($I_r^{\perp}(H_+\sigma_+\sigma_0H_0) < I_r^{\parallel}(H_+\sigma_+\sigma_0H_0)$).

Nagata [1970], summing up the results to his research, came to the conclusion that the piezoremanent magnetization was caused by the irreversible displacement of boundaries of 90° domains. This allowed him, together with Carleton [Nagata and Carleton, 1968, 1969], to construct the theory of piezoremanent magnetization, which is in a good agreement with the experiments described above. However, the proposed mechanism of piezoremanent magnetization is not unique. Thus, Dunlop et al. [1969], on the basis of a simplified expression for the critical field of the irreversible change in the magnetic moment of a single-domain grain $H_0(\sigma) = H_0(0) - 3\lambda_s\sigma/I_s$ (where I_s and λ_s are the spontaneous magnetization and isotropic magnetostriction constant), showed that the relationships between the different forms of piezoremanent magnetization presented above are also fulfilled for the system of single-domain particles. The model of single-domain uniaxial particles made it possible in the low stress approximation, when $3\lambda_s\sigma/I_s \ll H_0(0)$, to calculate different forms of piezoremanent magnetization [Afremov and Belokon', 1980a, 1980b].

However, the majority of magnetic minerals of rocks are multiaxial crystals, for example, containing magnetite or titanomagnetite. Moreover, the mechanical stresses that appear as a result of seismic processes may not satisfy the smallness condition. Therefore, we can quite naturally extend the model utilized in [Afremov and Belokon', 1980a, 1980b] to the multiaxial particles, as well as to determine the piezoremanent magnetization within the entire range of stresses.

1. EQUILIBRIUM STATES AND THE CRITICAL FIELD OF SINGLE-DOMAIN PARTICLE

Model

1. We consider an ensemble of N single-domain noninteracting magnetic particles incorporated in the nonmagnetic matrix, which have the shape of oblong ellipsoid.

2. The crystallographic symmetry of magnetic material is assumed to be cubic, and the axis, separated by the crystallographic anisotropy (axis $\langle k_A\rangle$), makes an angle α with the major axis of the ellipsoid.

3. The uniaxial mechanical stresses σ makes an angle β with axis $\langle k_A\rangle$ if they are applied in the plane in which lie the major axis of the ellipsoid and $\langle k_A\rangle$ axis, or the mechanical stresses σ are perpendicular to this plane.

4. The condition of magnetic uniaxiality is satisfied [Sholpo, 1970].

5. The magnetic moments of one-third of particles are considered to be oriented along the external magnetic field $\mathbf{H}$, and the remaining particles are perpendicular to it.

The equilibrium states of the magnetic moment of such particles can be determined by minimizing the free-energy density, which includes the energies of the magnetic moment of a particle in the field of mechanical stresses and the external magnetic field, as well as the energies of shape anisotropy and crystallographic anisotropy,

$$\begin{aligned} F = -\frac{1}{4}I_s^2\sin^2\vartheta\{3\tilde{\lambda}_{100}(1+\cos 2\varphi\cos 2\beta) \\ + 3\tilde{\lambda}_{111}\sin 2\varphi\sin 2\beta)\sigma \\ + k_N(1+\cos 2\alpha\cos 2\varphi + \sin 2\alpha\sin 2\varphi) \\ + k_A(1+\cos 2\varphi)\} - (\mathbf{H}, \mathbf{I}_s). \end{aligned} \tag{1}$$

Here, ϑ and φ are the polar coordinates of the vector of spontaneous magnetization $\vec{\mathbf{I}}_s$, counted from axes $\langle 100\rangle$ and $\langle 010\rangle$, respectively; $\tilde{\lambda}_{100} = \lambda_{100}/I_s^2$, $\tilde{\lambda}_{111} = \lambda_{111}/I_s^2$, λ_{100}, and λ_{111} are the magnetostriction constants; k_N is the shape anisotropy constant, equal to the difference in the demagnetizing factors along the short and long axes; k_A is the dimensionless constant of crystallographic anisotropy, $\mathbf{H} = \{H, \vartheta_H, \varphi_H\}$; and ϑ_H, and φ_H are the polar coordinates of vector $\mathbf{H}$. Relationship (1) can be converted as follows:

$$\begin{aligned} F = -\frac{1}{4}I_s^2\sin^2\vartheta \\ \times\{3\tilde{\lambda}_{100}\sigma + k_N + k_A + K\cos 2(\varphi-\delta)\} - (\mathbf{H}, \mathbf{I}_s). \end{aligned} \tag{2}$$

In the case when the mechanical stresses are parallel to the plane in which lie axis $\langle k_A\rangle$ and the major axis of the ellipsoid, the effective anisotropy constant K and angle δ, which assigns the position of effective axis relative to axis $\langle k_A\rangle$ parallel to axis $\langle 100\rangle$, are determined by the following relationships:

$$\begin{aligned} K = \sqrt{k_N^2 + 2k_NK_1\cos(2\alpha-\psi_0) + K_1^2}, \\ \tan 2\delta = \frac{k_N\sin 2\alpha + \tilde{\lambda}_2\sigma\sin 2\beta}{|k_A| + k_N\cos 2\alpha + \tilde{\lambda}_1\sigma\cos 2\beta}, \end{aligned} \tag{3}$$

$$\begin{aligned} \tilde{\lambda}_1 = \begin{cases} 3\tilde{\lambda}_{100}, & k_A > 0 \\ (\tilde{\lambda}_{100} + 5\tilde{\lambda}_{111})/2, & k_A < 0, \end{cases} \\ \tilde{\lambda}_2 = \begin{cases} 3\tilde{\lambda}_{111}, & k_A > 0 \\ (2\tilde{\lambda}_{100} + \tilde{\lambda}_{111}), & k_A < 0. \end{cases} \end{aligned} \tag{4}$$

$$\begin{aligned} K_1 = \sqrt{(|k_A| + \tilde{\lambda}_1\sigma\cos 2\beta)^2 + (\tilde{\lambda}_2\sigma\sin 2\beta)^2}, \\ \tan\psi_0 = \frac{\tilde{\lambda}_2\sigma\sin 2\beta}{|k_A| + \tilde{\lambda}_1\sigma\cos 2\beta}. \end{aligned} \tag{5}$$

In the second case (the mechanical stresses are applied perpendicularly to the plane that contains axis

$\langle k_A \rangle$ and the major axis of ellipsoid),

$$K = \sqrt{k_N^2 + 2k_N(|k_A| + \tilde{\lambda}_1\sigma)\cos 2\alpha + (|k_A| + \tilde{\lambda}_1\sigma)^2}, \quad (6)$$

$$\tan 2\delta = \frac{k_N \sin 2\alpha}{|k_A| + k_N \cos 2\alpha + \tilde{\lambda}_1\sigma}. \quad (7)$$

Analysis of free energy (2) shows that in the absence of magnetic field, the equilibrium positions of the magnetic moment of a grain coincide with the position of effective axis.

The critical field of the irreversible change in the magnetic moment of grain can be determined by minimizing expression (2). Regardless of sign of k_A, we have

$$H_0 = I_s K, \quad (8)$$

where the effective anisotropy constant K is determined by relationships (3)–(6). A specific feature of relationship (8) is nonmonotonic dependence of H_0 on the shape anisotropy. If $\sigma \parallel H$, then with an increase in k_N, the critical field reaches the minimum of $H_{0\min} = K_1 I_s \sin(2\alpha - \psi_0)$ when $k_N = -K_1\cos(2\alpha - \psi_0)$. In the case $\sigma \perp H$, the minimum of the critical field of $H_{0\min} = I_s(|k_A| + \tilde{\lambda}_1\sigma)\sin 2\alpha$ is attained when $k_N = -(|k_A| + \tilde{\lambda}_1\sigma)\cos 2\alpha$. Obviously, in the system of particles that have different elongation, the spectrum of critical fields should be distributed from $H_{0\min}$ to $H_{0\max} = I_s(k_A + k_N)$. In view of symmetry of expressions for the critical field (3), (6), and (8) with respect to k_N and $K_1 = K_1(\sigma)$, the dependence of H_0 on mechanical stresses is also nonmonotonic with respect to $K_1 = K_1(\sigma)$.

2. PIEZOREMANENT MAGNETIZATION

If we consider that the particles of ensemble are distributed throughout shape (k_N) and angles $\psi = 2\alpha$, then to each particle with critical field (8) on the plane (see Fig. 1), it is possible to assign a point with the polar coordinates (k_N, ψ). Then, the representative points of particles with the critical fields $H_0 \leqslant H$ fall into the range bounded in the diagram $\{k_N, \psi\}$ by a circle of radius H/I_s with the center at point $k_N = K_1$, $\psi = \pi - \psi_0$. Accordingly, in the second case ($\sigma \perp H$), the condition $H_0 \leqslant H$ is satisfied by the particles which fall in Fig. 2 into the region bounded by a circle of radius H/I_s with the center at point $k_N = k_A + \tilde{\lambda}_1\sigma$, $\psi = \pi$.

The particles whose representative points are presented in Fig. 1 determine the remanent magnetization $I_r^{\|}(\sigma_+H_+H_0\sigma_0)$. The magnetization $I_r^{\|}(\sigma_+H_+H_0\sigma_0)$ that arose in the field $H \leqslant K_1 I_s$, can be represented with the help of the following relationship:

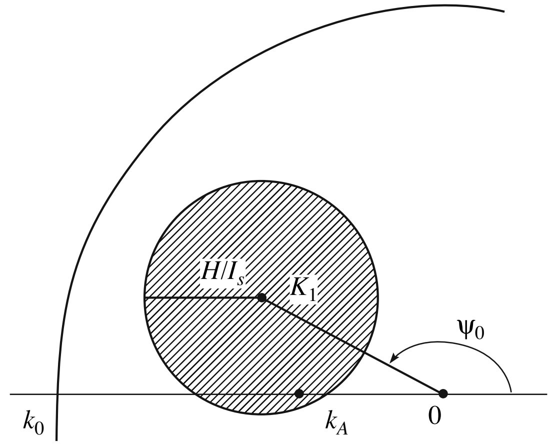

Fig. 1. Calculation of the longitudinal piezoremanent magnetization $I_r^{\|}(\sigma_+H_+H_0\sigma_0)$. See text for explanations.

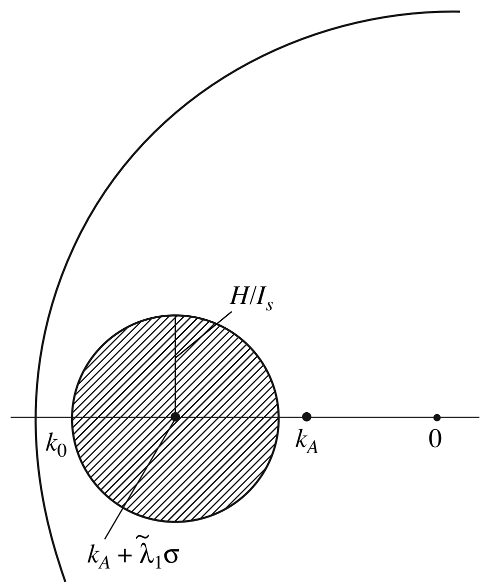

Fig. 2. Calculation of the transversal piezoremanent magnetization $I_r^{\perp}(\sigma_+H_+H_0\sigma_0)$. See text for explanations.

$$I_r^{\|}(\sigma_+H_+H_0\sigma_0) = \frac{I_s v}{V}\int_0^{\pi} d\beta \times \int_{\psi_0 - \arcsin X}^{\psi_0 + \arcsin X} \int_{K_1(-\cos(\psi-\psi_0) - \sqrt{X^2 - \sin^2(\psi-\psi_0)})}^{K_1(-\cos(\psi-\psi_0) + \sqrt{X^2 - \sin^2(\psi-\psi_0)})} f(k_N, \psi, \beta)\, dk_N = \frac{2cH^2}{3\pi^2 k_0 I_s}\int_0^{\pi} G(\pi/2, X)\frac{d\beta}{K_1}, \quad (9)$$

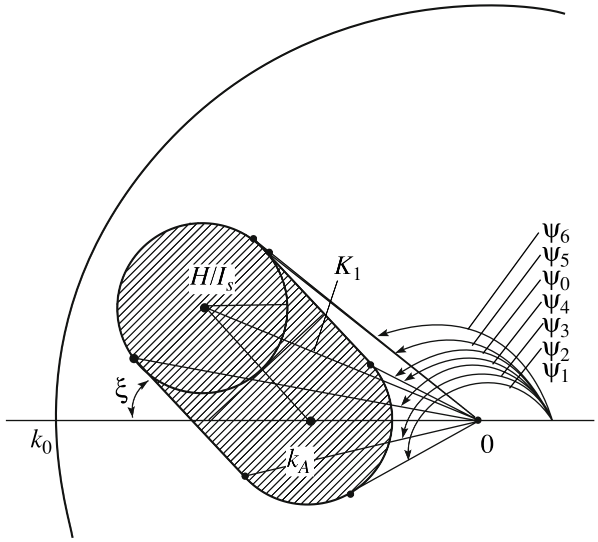

Fig. 3. Calculation of the longitudinal piezoremanent magnetization $I_r^{\parallel}(H_+\sigma_+\sigma_0 H_0)$. See text for explanations.

where v is the volume of particle, V is the volume of sample, $c = Nv/V$ is the bulk concentration of magnetic material, $G(\alpha, x) = F(\alpha, x) - (F(\alpha, x) - E(\alpha, z))/x^2$, $F(\alpha, x)$, $E(\alpha, x)$ are the elliptical integrals of the first and second kind, respectively, $X = H/K_1 I_s$, $f(k_N, \psi, \beta)$ represent the distribution function throughout k_N, ψ, and β, which is assumed to be uniform: $f(k_N, \psi, \beta) = N/6\pi^2 k_0$ if $0 \le k_N \le k_0$, $0 \le \psi \le 2\pi$, $0 \le \beta \le \pi$, and $f(k_N, \psi, \beta) = 0$ outside of the indicated interval. Relationship (9) can also be used to $I_r^{\parallel}(\sigma_+ H_+ H_0 \sigma_0)$, when $k_A < 0$, using the appropriate expression for K_1. In the case of low fields $H \ll k_A I_s$ and stresses $\tilde{\lambda}_1\sigma$, $\tilde{\lambda}_2\sigma \ll k_A$, the magnetization $I_r^{\parallel}(\sigma_+ H_+ H_0 \sigma_0)$ is proportional to the square of the field and it very weakly depends on the stresses regardless of the sign of k_A:

$$I_r^{\parallel}(\sigma_+ H_+ H_0 \sigma_0) = I_r(H)\left[1 - \frac{(\tilde{\lambda}_1\sigma)^2 + (\tilde{\lambda}_2\sigma)^2}{4k_A^2}\right]. \quad (10)$$

Here, $I_r(H) = I_r(H_+ H_0) = cH^2/6k_0 k_A I_s$ is the normal remanent magnetization.

Analogously, it is possible to conduct calculation of magnetization $I_r^{\parallel}(H_+\sigma_+\sigma_0 H_0)$ (see Fig. 3):

$$I_r^{\parallel}(H_+\sigma_+\sigma_0 H_0) = \frac{cI_s}{6\pi^2 k_0}$$

$$\times \int_0^{\pi} d\beta \left\{ \int_{\psi_1}^{\psi_2} d\psi \int_{k_{N_1}^{(1)}}^{k_{N_2}^{(1)}} dk_N + \int_{\psi_2}^{\psi_3} d\psi \int_{k_{N_1}^{(1)}}^{k_{N_2}^{(2)}} dk_N + \int_{\psi_3}^{\psi_4} d\psi \int_{k_{N_1}^{(1)}}^{k_{N_2}^{(1)}} dk_N \right.$$

$$\left. + \int_{\psi_3}^{\psi_4} d\psi \int_{k_{N_1}^{(1)}}^{k_{N_2}^{(2)}} dk_N \right\} = \frac{cI_s}{6\pi^2 k_0} \int_0^{\pi} d\beta$$

$$\times \Big\{ k_A x^2 [G(\alpha(\psi_3), x) - G(\alpha(\psi_1), x)]$$

$$+ k_A(\sin\psi_3 - \sin\psi_2) + K_1 X^2 [2G(\alpha(\psi_6 - \psi_0), X) \quad (11)$$

$$- G(\alpha(\psi_5 - \psi_0), X) - G(\alpha(\psi_4 - \psi_0), X)]$$

$$+ K_1(\sin(\psi_4 - \psi_0) - \sin(\psi_5 - \psi_3)) + \frac{1}{2}k_A(\sin\xi - x)$$

$$\times \ln\left[\frac{\theta(\psi_2)\theta(\psi_5)}{\theta(\psi_3)\theta(\psi_4)}\right] + k_A x \ln\left[\frac{\theta(\psi_3)}{\theta(\psi_4)}\right] \Big\},$$

$$x = H/k_A I_s, \quad X = H/K_1 I_s,$$

where the relationships $k_{N_{1,2}}^{(2)}(\psi) = K_1(-\cos(\psi - \psi_0) \mp \sqrt{X^2 - \sin^2(\psi - \psi_0)})$ and $k_{N_{1,2}}^{(1)}(\psi) = k_A(-\cos\psi \mp \sqrt{x^2 - \sin^2\psi})$ are the equations of circles in polar coordinates; $k_N^{(1)} = k_A(\sin\xi - x)/\sin(\psi - \xi)$ and $k_N^{(2)} = k_A(\sin\xi + x)/\sin(\psi - \xi)$ determine the equations of tangents to these circles (see Fig. 3); and $\alpha(\psi) = \arcsin(\sin\psi/x)$ and ψ_i are the solutions of the following equations: $\sin\psi_1 = x$, $k_{N_2}^{(1)}(\psi_2) = k_{N_2}^{(2)}(\psi_2)$, $k_{N_1}^{(1)}(\psi_3) = k_{N_1}^{(1)}(\psi_3)$, $k_{N_2}^{(2)}(\psi_4) = k_{N_2}^{(2)}(\psi_4)$, $k_{N_1}^{(2)}(\psi_5) = k_{N_1}^{(1)}(\psi_5)$, $\sin\psi_6 = X$, $\tan\xi = (\tilde{\lambda}_2/\tilde{\lambda}_1)\tan 2\beta$, $\theta(\psi) = (1 + \cos\psi)/(1 - \cos\psi)$.

In the approximation $H \ll k_A I_s$ and $\tilde{\lambda}_1\sigma$, $\tilde{\lambda}_2\sigma \ll k_A$, expression (11) can be reduced to the form

$$I_r^{\parallel}(H_+\sigma_+\sigma_0 H_0) = I_r^{\parallel}(\sigma_+ H_+ H_0 \sigma_0) + \frac{c\Lambda|\sigma|H}{3\pi k_0 |k_A|}, \quad (12)$$

$$\Lambda = \frac{4|\tilde{\lambda}_1|}{\pi} \int_0^{\frac{\pi}{2}} \sqrt{1 + \left[\left(\frac{\tilde{\lambda}_2}{\tilde{\lambda}_1}\right)^2 - 1\right]\sin^2\eta}\, d\eta$$

$$= \begin{cases} \dfrac{4|\tilde{\lambda}_1|}{\pi} E\left(\dfrac{\pi}{2}, \sqrt{1 - \left(\dfrac{\tilde{\lambda}_2}{\tilde{\lambda}_1}\right)^2}\right), & |\tilde{\lambda}_2| < |\tilde{\lambda}_1| \\ \dfrac{4|\tilde{\lambda}_2|}{\pi} E\left(\dfrac{\pi}{2}, \sqrt{1 - \left(\dfrac{\tilde{\lambda}_1}{\tilde{\lambda}_2}\right)^2}\right), & |\tilde{\lambda}_2| > |\tilde{\lambda}_1|. \end{cases} \quad (13)$$

It is obvious that in contrast to $I_r^{\parallel}(\sigma_+H_+H_0\sigma_0)$, the piezoremanent magnetization $I_r^{\parallel}(H_+\sigma_+\sigma_0H_0)$ generated in low fields ($H \ll |\tilde{\lambda}_1\sigma|I_s$, $|\tilde{\lambda}_2\sigma|I_s$) is almost proportional to H and σ; moreover $I_r^{\parallel}(\sigma_+H_+H_0\sigma_0) \ll I_r^{\parallel}(H_+\sigma_+\sigma_0H_0)$.

If stresses are applied perpendicularly to the plane containing the axis of crystallographic anisotropy and the major axis of the ellipsoid and if $k_A > 0$, then, the particles whose representative points fall into the shaded region in Fig. 2 will contribute to the magnetization $I_r^{\perp}(\sigma_+H_+H_0\sigma_0)$:

$$I_r^{\perp}(\sigma_+H_+H_0\sigma_0) = \frac{cI_s}{3\pi k_0} \times \int\limits_{\pi-\arcsin X}^{\pi+\arcsin X} \int\limits_{(k_A+\tilde{\lambda}_1\sigma)(-\cos\psi-\sqrt{X^2-\sin^2\psi})}^{(k_A+\tilde{\lambda}_1\sigma)(-\cos\psi+\sqrt{X^2-\sin^2\psi})} dk_N = \frac{2cH^2}{3\pi k_0(k_A+\tilde{\lambda}_1\sigma)I_s}G(\pi/2, X), \quad (14)$$

$$X = \frac{H}{(k_A+\tilde{\lambda}_1\sigma)I_s}.$$

In the approximation of low fields $H \ll k_AI_s$ and stresses $\tilde{\lambda}_1\sigma$, $\tilde{\lambda}_2\sigma \ll k_A$, relationship (14) can be reduced to the following form:

$$I_s^{\perp}(\sigma_+H_+H_0\sigma_0) = I_r(H)\left(1-\frac{\tilde{\lambda}_1\sigma}{|k_A|}\right). \quad (15)$$

Calculation of $I_s^{\perp}(\sigma_+H_+H_0\sigma_0)$ is analogous to the calculation of $I_r^{\parallel}(H_+\sigma_+\sigma_0H_0)$ given above. In the case when $H \leq k_AI_s$ and $k_A > 0$, in the $\{k_N, \psi\}$ diagram (see Fig. 4), the shaded region corresponds to remanent magnetization $I_r^{\perp}(H_+\sigma_+\sigma_0H_0)$. Therefore, we will have

$$I_r^{\perp}(\sigma_+H_+H_0\sigma_0) = \frac{cI_s}{6\pi k_0}\left\{\int\limits_{\pi-\psi_1}^{\pi-\psi_2} d\psi \int\limits_{k_A(-\cos\psi-\sqrt{x^2-\sin^2\psi})}^{H/(I_s\sin\psi)} dk_N + \int\limits_{\pi-\psi_2}^{\pi} d\psi \int\limits_{k_A(-\cos\psi+\sqrt{x^2-\sin^2\psi})}^{(k_A+\tilde{\lambda}_1\sigma)(-\cos\psi+\sqrt{X^2-\sin^2\psi})} dkN\right\} \quad (16)$$

$$= \frac{cI_sk_A}{3\pi k_0}\left\{x\ln\frac{\theta(\psi_2)}{\theta(\psi_1)} + G(\alpha(\psi_1), x) + \left[1-\frac{\tilde{\lambda}_1\sigma}{k_A}\right]G(\alpha(\psi_2), X) - \sin\psi_1 + \left[1+\frac{\tilde{\lambda}_1\sigma}{k_A}\right]\sin\psi_2\right\},$$

where $X = H/(k_A + \tilde{\lambda}_1\sigma)I_s$.

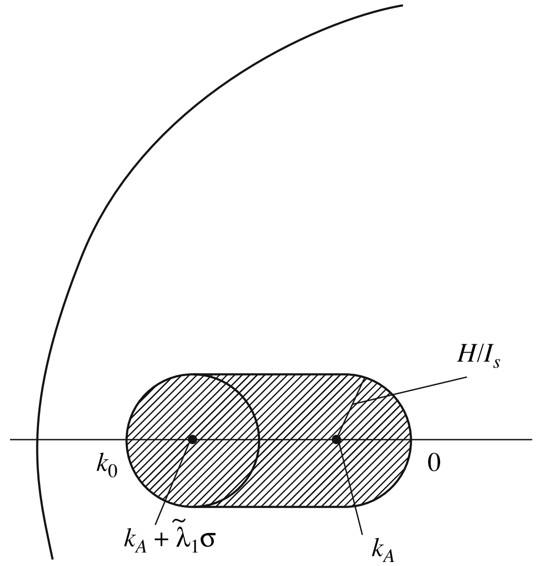

Fig. 4. Calculation of the transversal piezoremanent magnetization $I_r^{\perp}(H_+\sigma_+\sigma_0H_0)$. See text for explanations.

In low fields ($H \ll k_AI_s$) and with the stresses of $\tilde{\lambda}_1\sigma$, $\tilde{\lambda}_2\sigma \ll k_A$, relationship (16) can be reduced to the form

$$I_r^{\perp}(H_+\sigma_+\sigma_0H_0) = I_r^{\perp}(\sigma_+H_+H_0\sigma_0) + \frac{c|\tilde{\lambda}_1\sigma|H}{3\pi k_0|k_A|}. \quad (17)$$

It is obvious that relationship (17), just like (12), depends on the sign of the constant of crystallographic anisotropy.

3. RESULTS AND DISCUSSION

Our enables makes it possible to distinguish two basic kinds of piezoremanent magnetization of the system of single-domain particles, which differ in the mechanism of formation. The magnetization of the first kind, $I_r(\sigma_+H_+H_0\sigma_0)$, which arises in the presence of mechanical stresses σ, is equivalent to the normal remanent magnetization $I_r(H_+H_0)$. The mechanism of formation of the piezoremanent magnetization of the second kind, $I_r(H_+\sigma_+\sigma_0H_0)$, which is generated when there is a change in σ, is connected with the nonmonotonic behavior of the critical fields of particles. Obviously, precisely this kind of piezoremanent magnetization mainly determines the piezomagnetic effect.

For the comparison with the experiment we will use the works [Nagata and Carleton, 1968, 1969; Nagata, 1970]. In these works, the properties of the piezorema-

Dependence of the ratio of the longitudinal and transversal piezoremanent magnetizations $I_r(H_+, \sigma_+, \sigma_0, H_0)$, the constant of proportionality of piezoremanent magnetization $C^{\parallel}$, and sensitivity of remanent saturation magnetization β_{rs} of titanomagnetites $Fe_{3-x}Ti_xO_4$ on titanium concentration x. The maximum values of coefficients correspond to $k_0 = 1$, the minimum values correspond to $k_0 = 4\pi$. The values of I_s, k_A, λ_{100}, and λ_{111} utilized in the calculations are borrowed from the work [Syono, 1965]

x	0	0.04	0.1	0.18	0.3	0.56
$I_r^{\parallel}(H_+\sigma_+\sigma_0H_0)/I_r^{\perp}(H_+\sigma_+\sigma_0H_0)$	1.34	1.40	1.53	1.66	1.81	2.38
$C^{\parallel}_{\max}$, 10^{-4} CGS	2.2	1.9	1.8	3.2	3.6	13
$C^{\parallel}_{\min}$, 10^{-4} CGS	0.2	0.2	0.1	0.3	0.3	1
$\beta_{rs\,\max}$, 10^{-9} Pa^{-1}	0.5	0.9	1.2	3.0	4.3	23
$\beta_{rs\,\min}$, 10^{-9} Pa^{-1}	0.06	0.13	0.21	0.5	0.9	6

nent magnetization formed in the weak fields (1–20 Oe) with the relatively small longitudinal or transversal compression (less than 120 kgf/cm^2) are most thoroughly studied. The experimental fields and stresses satisfy the approximation of low fields ($H \ll k_AI_s$) and stresses ($\tilde{\lambda}_1\sigma$, $\tilde{\lambda}_2\sigma \ll k_A$), utilized in the present work. Indeed, assuming for magnetite $|k_A| \approx 0.6$ ($k_A < 0$), I_s = 480 CGSM, $\lambda_{100} = -2.0 \times 10^{-5}$, $\lambda_{111} = 7.8 \times 10^{-5}$ [Syono, 1965], we have: $k_AI_s \approx 240$ Oe, $\tilde{\lambda}_1\sigma \approx 0.09$, $\tilde{\lambda}_2\sigma \approx 0.04$ (with σ = 120 kgf/cm^2).

In this approximation, the piezoremanent magnetization $I_r(\sigma_+H_+H_0\sigma_0)$, regardless of the field and stress orientation, depends quadratically on the field ($I_r(\sigma_+H_+H_0\sigma_0) \sim H^2$) and weakly depends on σ. Moreover, in contrast to the results of [Afremov and Belokon', 1980b; Afremov and Panov, 2004] and [Nagata and Carleton, 1969], the increase in the stresses parallel to the field, regardless of their sign and the sign of the constant of crystallographic anisotropy, should lead to a decrease in magnetization $I_r^{\parallel}(\sigma_+H_+H_0\sigma_0)$, which agrees with the experimental data in [Nagata and Carleton, 1968]. The effect of stresses perpendicular to the field under compression should increase the rate of growth of $I_r^{\perp}(\sigma_+H_+H_0\sigma_0)$ by a factor of $(1 + |\tilde{\lambda}_1\sigma|/k_A)$ and decrease it by a factor of $(1 - |\tilde{\lambda}_1\sigma|/k_A)$ under extension. It should be noted that the ratio of increments ($\Delta I_r(\sigma_+H_+H_0\sigma_0) = I_r(\sigma_+H_+H_0\sigma_0) - I_r(H_+H_0)$ of the piezoremanent magnetizations caused by weak effects parallel and perpendicular with respect to a low field H, regardless of the sign of the constant of crystallographic anisotropy, is determined by the following relationship: $\Delta I_r^{\parallel}(\sigma_+H_+H_0\sigma_0)/\Delta I_r^{\perp}(\sigma_+H_+H_0\sigma_0) \ll 1$, which is differed from that obtained using the model of multidomain particles [Nagata and Carleton, 1969] $\Delta I_r^{\parallel}(\sigma_+H_+H_0\sigma_0)/\Delta I_r^{\perp}(\sigma_+H_+H_0\sigma_0) = 2$. Moreover, for titanomagnetites ($k_A < 0$), magnetizations formed in the presence of axial compression obey the following inequality $I_r^{\perp}(\sigma_+H_+H_0\sigma_0) < I_r^{\parallel}(\sigma_+H_+H_0\sigma_0)$, which agrees with the experiment of [Nagata and Carleton, 1968]. If $k_A > 0$, then the given relationship is fulfilled only with $\lambda_{100} > 0$, and the inverse relationship between magnetizations corresponds to negative values of λ_{100}.

In low fields ($H \ll \tilde{\lambda}_1\sigma I_s$, $\tilde{\lambda}_2\sigma I_s$), the magnetizations $I_r^{\parallel}(H_+\sigma_+\sigma_0H_0)$ and $I_r^{\perp}(H_+\sigma_+\sigma_0H_0)$, just like in the theory [Nagata and Carleton, 1969], are proportional to the field and stresses and their ratio $I_r^{\parallel}(H_+\sigma_+\sigma_0H_0)/I_r^{\perp}(H_+\sigma_+\sigma_0H_0) = \Lambda/\tilde{\lambda}_1$ depends on the magnetostriction constants and the sign of the constant of crystallographic anisotropy. This result differs from that obtained in the model [Nagata and Carleton, 1969] within the framework of which the ratio $I_r^{\parallel}(H_+\sigma_+\sigma_0H_0)/I_r^{\perp}(H_+\sigma_+\sigma_0H_0) = 4/3$, which does not agree with the results of experiment [Nagata and Carleton, 1968], according to which this ratio $I_r^{\parallel}(H_+\sigma_+\sigma_0H_0)/I_r^{\perp}(H_+\sigma_+\sigma_0H_0) \approx 1.1$–$1.4$. Calculation of the ratio of magnetizations for titanomagnetites shows that depending on the titanium concentration $I_r^{\parallel}(H_+\sigma_+\sigma_0H_0)/I_r^{\perp}(H_+\sigma_+\sigma_0H_0) \approx 1.34$–$2.38$ (see table).

If condition $H \ll |\tilde{\lambda}_{100}\sigma| I_s$ is not satisfied, then

$$I_r^{\parallel}(H_+\sigma_+\sigma_0H_0)/I_r^{\perp}(H_+\sigma_+\sigma_0H_0) = 1 + (\Lambda/\tilde{\lambda}_1 - 1)/(1 + \pi H/2\tilde{\lambda}_1\sigma I_s),$$

and for magnetite, when $k_A > 0$, depending on the stresses and magnetic field, the ratio $I_r^{\parallel}(H_+\sigma_+\sigma_0H_0)/I_r^{\perp}(H_+\sigma_+\sigma_0H_0)$ can vary from 1 to 5.2 and the exceeding of $I_r^{\parallel}(H_+\sigma_+\sigma_0H_0)$ above $I_r^{\perp}(H_+\sigma_+\sigma_0H_0)$ can vary by a factor of 0 to 4.5, respectively. If $k_A < 0$, then $I_r^{\parallel}(H_+\sigma_+\sigma_0H_0)/I_r^{\perp}(H_+\sigma_+\sigma_0H_0)$ takes values from 1 to 1.34, and $I_r^{\parallel}(H_+\sigma_+\sigma_0H_0)$ can exceed $I_r^{\perp}(H_+\sigma_+\sigma_0H_0)$ up to 34%, which agrees with the results of measurements [Nagata and Carleton, 1968; Nagata, 1970] (10–35%).

It should be noted that the rate of variation of $I_r^{\|}(H_+\sigma_+\sigma_0 H_0) \quad - C^{\|}(\sigma) = I_r^{\|}(H_+\sigma_+\sigma_0 H_0)/H = (c|\sigma|(3\pi k_0 k_A))\Lambda$, calculated for 116 kgf/cm^2 and concentration $c = 10^{-2}$, depending on elongation k_0, which can vary within the range from 1 to 4π, depending on titanium concentration in titanomagnetites, are distributed within the range (0.2–2.2) $\times$ 10^{-4} CGS for magnetite and (1–13) $\times$ 10^{-4} CGS for titanomagnetite with x = 0.56 (see the table). These values are in agreement with the measurements carried out with the samples of igneous rocks: $C_{\text{exper}}^{\|} = (0.2–10) \times 10^{-4}$ CGS [Nagata, 1970].

Taking into account the correlation between the rate of change in the piezoremanent magnetization $C^{\|}$ and the sensitivity to stress β, it is possible to justify the assumption of [Nishida et al., 2004] on values of $\beta \sim 10^{-8}$ Pa^{-1} that are higher than those presented in [Stacey and Johnston, 1972], $\beta \sim 10^{-9}$ Pa^{-1}, utilized for modeling the piezomagnetic effect. The high values of sensitivity to stresses can be connected with the high concentration of titanium in the rock. This assumption can be confirmed by calculation of sensitivity to the stress for the remanent saturation magnetization, $\beta_{rs} = \tilde{\lambda}_2/2(k_A + k_0)$, which is presented in the table.

The relationships obtained in the present work are valid for any values of stresses and in the approximation $\tilde{\lambda}_1\sigma$, $\tilde{\lambda}_2\sigma \ll k_A$, include the results of [Afremov and Belokon', 1980b] if in formulas (12), (15), and (17) we replace $\tilde{\lambda}_1$ with $\hat{\lambda}_1 - \hat{\lambda}_2$, $\tilde{\lambda}_2$ with $\hat{\lambda}_4$, $k_A + k_N$ with $k_A + k_N + (\hat{\lambda}_1 + \hat{\lambda}_2)\sigma$, where $\tilde{\lambda}_i = \lambda_i/I_s^2$, λ_i are four magnetostriction constants of the uniaxial crystal.

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