Distribution of Electric Field Strength in Optical Planar Waveguides with Nanoparticles

Andrey V. Panov*

Abstract: *A symmetrical optical planar waveguide containing a layer with nanoparticles (the active layer) is examined by virtue of numerical simulation. We calculate the dependence of the electric strength field amplitude on the volume concentration of the nanoparticles within the active layer. It is shown that even TE-modes are most suitable for observation of nonlinear optical effects.*

Keywords: *planar waveguide, nonlinear optical interaction*

In recent years much interest has been centered around a study of the nonlinear optical phenomena, which are observed in the media, containing nanoparticles[1, 2]. Such the medium with nanoparticles can be embedded, for example, into planar waveguides where the nonlinear optical interaction can occur[1]. It is of great interest to know the light intensity in the medium with nanoparticles since nonlinear optical effects depend on the strength the of electric field.

Let us examine the depicted in Fig. 1 symmetrical three-layered planar waveguide, which is contained from two sides in the substrate (regions 0 and 4 in the figure). We will assume that basic waveguide layers are regions 1 and 3 with thickness d_1 , region 2 with thickness d_2 ($d_2 =\ d_1$) consists of the medium with the nanoparticles, in which nonlinear optical interaction occurs. We will consider that the particle sizes are much lower than the wavelength of light.

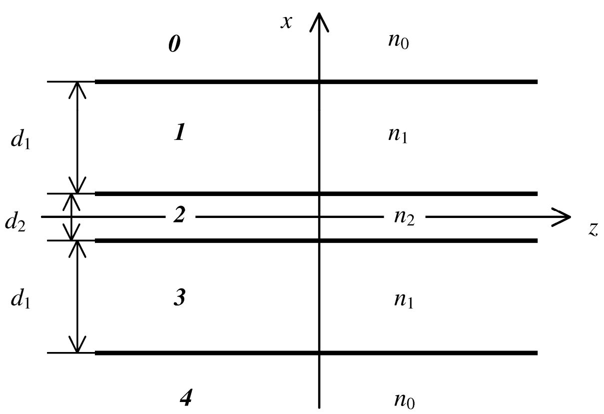


Fig. 1. A sketch of the planar waveguide.

* *Ph.D., Senior Researcher, Institute of Automation and Control Processes, FEB RAS, Russia*
E-mail: panov@iacp.dvo.ru

The distributions of vectors of electric an magnetic strength $\mathbf{E}$, $\mathbf{H}$ in the waveguide are found by the means of the solution of Maxwell's equations. Assuming that light propagation in the waveguide does occur in the direction of z -axis, we will seek the solutions in the form

$$\mathbf{E}, \mathbf{H} \propto \exp[i(\omega t - \beta z)], \tag{1}$$

where β is longitudinal propagation number, which depends on the number of mode, ω is cyclic frequency, t is the time.

It is common knowledge that the solutions of Maxwell's equations for the planar waveguides are decomposed into two types: H -waves (TE-modes, $H_z, H_x, E_y \neq 0$) and E -waves (TH-modes, $E_z, E_x, H_y \neq 0$)[3].

At first let us study E -waves (TH-modes). Only one of three components of the vectors $\mathbf{E}$ and $\mathbf{H}$ is independent, for example H_y . In this case Maxwell's equations are reduced to the equation

$$\frac{\partial^2 H_y}{\partial x^2} + (n(x)^2 k^2 - \beta^2) H_y = 0, \tag{2}$$

where $n(x)$ is the refractive index of medium, k is the wave number in the vacuum. E_z , E_x are expressed as H_y as follows:

$$E_{x,j} = \frac{\beta}{kn_j} H_{y,j}, \quad E_{z,j} = -\frac{i}{kn_j} \frac{\partial H_{y,j}}{\partial x}, \tag{3}$$

where j is the region number (see Fig. 1). In the case $n_2 > n_1$ (the effective layer is waveguiding) we can write the solutions of the equation (2) in the form:

$$H_{y,0} = A_0 \exp(-\alpha_0 x), \tag{4}$$

$$H_{y,1} = A_1 \cos(k_{1,x}x) + B_1 \sin(k_{1,x}x), \quad (5)$$

$$H_{y,2} = A_2 \cos(k_{2,x}x) + B_2 \sin(k_{2,x}x), \quad (6)$$

$$H_{y,3} = A_3 \cos(k_{1,x}x) + B_3 \sin(k_{1,x}x), \quad (7)$$

$$H_{y,4} = A_4 \exp(\alpha_0 x), \quad (8)$$

$$k_{1,x} = \sqrt{n_1^2 k^2 - \beta^2}, \quad k_{2,x} = \sqrt{n_2^2 k^2 - \beta^2}, \quad \alpha_0 = \sqrt{\beta^2 - n_0^2 k^2}.$$

Integration constants A_j, B_j are sought with the boundary conditions, which can be written for E_z and H_y in the form:

$$A_0 \exp(-\alpha_0 (d_1 + d_2/2)) = A_1 \cos(k_{1,x}(d_1 + d_2/2)) + B_1 \sin(k_{1,x}(d_1 + d_2/2)), \quad (9)$$

$$A_1 \cos(k_{1,x}d_2/2) + B_1 \sin(k_{1,x}d_2/2) = A_2 \cos(k_{2,x}d_2/2) + B_2 \sin(k_{2,x}d_2/2), \quad (10)$$

$$A_2 \cos(k_{2,x}d_2/2) - B_2 \sin(k_{2,x}d_2/2) = A_3 \cos(k_{1,x}d_2/2) - B_3 \sin(k_{1,x}d_2/2), \quad (11)$$

$$A_3 \cos(k_{1,x}(d_1 + d_2/2)) - B_3 \sin(k_{1,x}(d_1 + d_2/2)) = A_4 \exp(-\alpha_0 (d_1 + d_2/2)), \quad (12)$$

$$A_0 \alpha_0 \exp\left\{-\alpha_0 \left(d_1 + d_2/2\right)\right\} / n_0^2 = \frac{k_{1,x}}{n_1^2} \left\{ A_1 \sin\left(k_{1,x}\left(d_1 + d_2/2\right)\right) - B_1 \cos\left(k_{1,x}\left(d_1 + d_2/2\right)\right) \right\}, \quad (13)$$

$$\frac{k_{1,x}}{n_1^2} \left\{ A_1 \sin\frac{k_{1,x}d_2}{2} - B_1 \cos\frac{k_{1,x}d_2}{2} \right\} = \frac{k_{2,x}}{n_2^2} \left\{ A_2 \sin\frac{k_{2,x}d_2}{2} - B_2 \cos\frac{k_{2,x}d_2}{2} \right\}, \quad (14)$$

$$\frac{k_{2,x}}{n_2^2} \left\{ A_2 \sin\frac{k_{2,x}d_2}{2} + B_2 \cos\frac{k_{2,x}d_2}{2} \right\} = \frac{k_{1,x}}{n_1^2} \left\{ A_3 \sin\frac{k_{1,x}d_2}{2} + B_3 \cos\frac{k_{1,x}d_2}{2} \right\}, \quad (15)$$

$$\frac{k_{1,x}}{n_1^2} \left\{ A_3 \sin\left(k_{1,x}\left(d_1 + d_2/2\right)\right) + B_3 \cos\left(k_{1,x}\left(d_1 + d_2/2\right)\right) \right\} = \frac{\alpha_0}{n_0^2} A_4 \exp\left\{-\alpha_0 \left(d_1 + d_2/2\right)\right\}. \quad (16)$$

The set of algebraic equations (9)–(16) makes it possible to express $A_1 \ldots A_4$, B_j through the constant A_0, which is preset by the intensity of the light introduced into the waveguide, hereinafter for simplicity we will assume $A_0 = 1$. Let us write the set of equations (9)–(16) with respect to A_j, B_j in the matrix form and after making the determinant of the left side of the matrix equation equal to zero we will obtain transcendental equation for β:

$$\left\{ \frac{k_{1,x}^3}{n_1^4} - \frac{2k_{1,x}^2 k_{2,x}}{n_1^2 n_2^2} + \frac{k_{1,x} k_{2,x}^2}{n_2^4} \right\} \frac{\alpha_0}{n_0^2 n_1^2} \cos(2d_1 k_{1,x} - d_2 k_{2,x}) - \left\{ \frac{k_{1,x}^3}{n_1^4} + \frac{2k_{1,x}^2 k_{2,x}}{n_1^2 n_2^2} + \frac{k_{1,x} k_{2,x}^2}{n_2^4} \right\} \frac{\alpha_0}{n_0^2 n_1^2} \cos(2d_1 k_{1,x} + d_2 k_{2,x}) + \left\{ \left[\frac{k_{2,x}^2}{n_2^4} - \frac{k_{1,x}^2}{n_1^4} \right] \frac{\alpha_0^2}{n_0^4} - \frac{k_{1,x}^4}{n_1^8} + \frac{k_{1,x}^2 k_{2,x}^2}{n_1^4 n_2^4} \right\} \sin(k_{2,x} d_2) + \frac{1}{2} \left\{ \frac{k_{1,x}^4}{n_1^8} + \frac{2k_{1,x}^3 k_{2,x}}{n_1^6 n_2^2} + \frac{k_{1,x}^2 k_{2,x}^2}{n_1^4 n_2^4} - \left[\frac{k_{1,x}^2}{n_1^4} + \frac{2k_{1,x} k_{2,x}}{n_1^2 n_2^2} + \frac{k_{2,x}^2}{n_2^4} \right] \frac{\alpha_0^2}{n_0^4} \right\} \sin(2k_{1,x} d_1 + k_{2,x} d_2) + \frac{1}{2} \left\{ \left[\frac{k_{1,x}^2}{n_1^4} - \frac{k_{1,x} k_{2,x}}{n_1^2 n_2^2} + \frac{k_{2,x}^2}{n_2^4} \right] \frac{\alpha_0^2}{n_0^4} - \frac{k_{1,x}^4}{n_1^8} + \frac{2k_{1,x}^3 k_{2,x}}{n_1^6 n_2^2} - \frac{k_{1,x}^2 k_{2,x}^2}{n_1^4 n_2^4} \right\} \sin(2k_{1,x} d_1 - k_{2,x} d_2), \quad (17)$$

which is solved numerically.

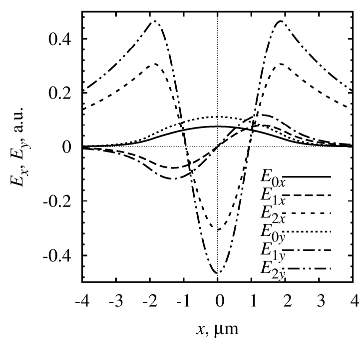


Fig. 2. The magnitude of E_x of TH-modes and E_y of TE-modes versus x coordinate for the homogeneous waveguide ($n_2 = n_1 = 1.52$).

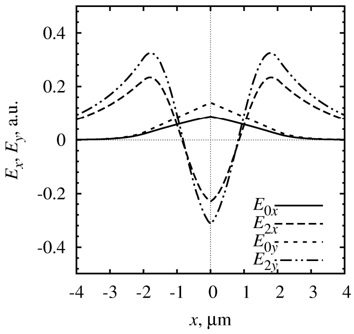


Fig. 3. The magnitude of E_x of TH-modes and E_y of TE-modes versus x coordinate for the waveguide with $p = 0.2$, $n_m = 1.6$, $n_p = 2.5$.

Numerical simulations were carried out for the waveguides with the refractive indices $n_0 = 1.51$, $n_1 = 1.52$ and with the thicknesses $d_2 = 2\,\mu$ m and $d_2 = 0.01\,\mu$ m. The effective refractive index of active medium with the nanoparticles can be calculated with the aid of the formula, obtained by Bruggeman in the model of the effective medium[4]:

$$\frac{p(\varepsilon_p - \varepsilon_e)}{\varepsilon_p + 2\varepsilon_e} + \frac{(1-p)(\varepsilon_m - \varepsilon_e)}{\varepsilon_m + 2\varepsilon_e} = 0, \qquad (18)$$

where $\varepsilon_p = n_p^2$ is the dielectric constant of nanoparticles, $\varepsilon_m = n_m^2$ is the dielectric constant of the medium, which contains nanoparticles, $\varepsilon_e = n_2^2$ is the effective dielectric constant of the central layer of the waveguide, p is volume concentration of nanoparticles in the central layer. For modeling the waveguiding active layer we set $n_p = 2.5$ which corresponds to TiO_2.

Figures 2, 3 show the results of the calculations of dependencies of the x component of the electric field strength amplitude on x for TH-modes (E_x curves). Let us note, the strength of the transverse component of the electric field on the graphs is expressed in the dimensionless units, connected with the excitation of the specific mode in the waveguide (arbitrariness of the selection of constant A_0). Fig. 2 depicts the distribution of E_x in the homogeneous waveguide ($n_2 = n_1 = 1.52$), figure 3 illustrates the distribution of E_x in the waveguides with the active layer. We can see from the figure, an increase in n_p leads to the larger concentration of the electric field strength of the zero TH-modes in the active layer. For the odd mode in the center of waveguide, as one would expect for the symmetrical waveguides, E_x vanishes at $x = 0$.

In the case, when the active layer of the waveguide has the less optical density ($n_2 < n_1$) the part of the equations for finding the value of electric field will be written by other means. Formula (6) must be replaced by

$$H_{y,2} = A_2 \exp(k_{2,x} x) + B_2 \exp(k_{2,x} x), \qquad (19)$$

accordingly the equations for the boundary conditions (10), (11), (14), (15) by

$$\begin{aligned} &A_1 \cos(k_{1,x} d_2/2) + B_1 \sin(k_{1,x} d_2/2) = \\ &A_2 \exp(k_{2,x} d_2/2) + B_2 \exp(-k_{2,x} d_2/2), \end{aligned} \qquad (20)$$

$$\begin{aligned} &A_2 \exp(-k_{2,x} d_2/2) + B_2 \exp(k_{2,x} d_2/2) = \\ &A_3 \cos(k_{1,x} d_2/2) - B_3 \sin(k_{1,x} d_2/2), \end{aligned} \qquad (21)$$

$$\begin{aligned} &\frac{k_{1,x}}{n_1^2}\left\{A_1 \sin\frac{k_{1,x} d_2}{2} - B_1 \cos\frac{k_{1,x} d_2}{2}\right\} = \\ &\frac{k_{2,x}}{n_2^2}\left\{B_2 \exp\left[-\frac{k_{2,x} d_2}{2}\right] - A_2 \exp\frac{k_{2,x} d_2}{2}\right\}, \end{aligned} \qquad (22)$$

$$\begin{aligned} &\frac{k_{2,x}}{n_2^2}\left\{A_2 \exp\left[-\frac{k_{2,x} d_2}{2}\right] - B_2 \exp\frac{k_{2,x} d_2}{2}\right\} = \\ &\frac{k_{1,x}}{n_1^2}\left\{A_3 \sin\frac{k_{1,x} d_2}{2} + B_3 \cos\frac{k_{1,x} d_2}{2}\right\}. \end{aligned} \qquad (23)$$

In this case also we should change the expression for $k_{2,x}$:

$$k_{2,x} = \sqrt{\beta^2 - n_2^2 k^2}. \qquad (24)$$

Accordingly the characteristic equation for finding the longitudinal propagation number β changes. The results of calculated distribution in the waveguide are depicted in Fig. 4. Let us note that in this case E_x of even modes is ``extruded'' from the central layer, since the electromagnetic wave is damped in it. In particular higher modes have an explicit minimum at $x = 0$.

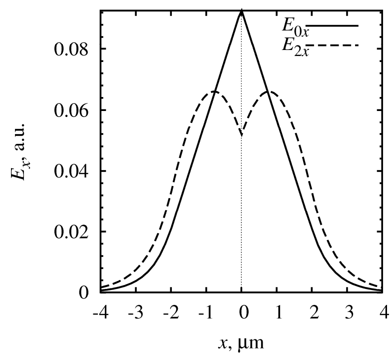


Fig. 4. The magnitude of E_x of TH-modes versus x coordinate for the waveguide with $n_2 = 1.2$.

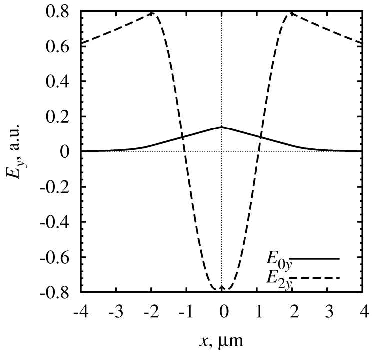
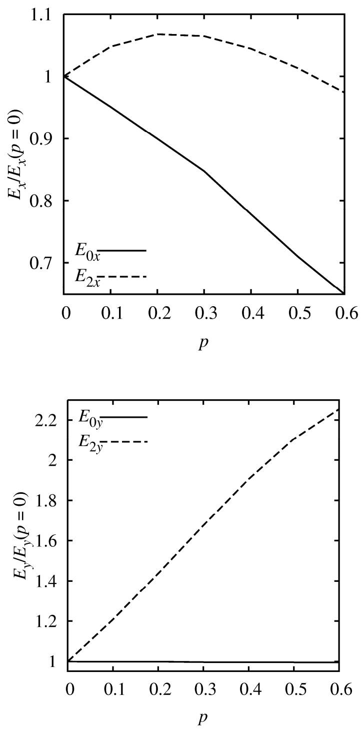


Fig. 5. The magnitude of E_y of TE-modes versus x coordinate for the waveguide with $n_2 = 1.2$.

The solutions in the case of H -waves (TE-modes) are analogous given above, the results of calculations are illustrated in figures 2, 3, 5 (E_y curves). The results qualitatively repeat the profiles of the transverse component of vector $\mathbf{E}$ of the TH-modes.



(*a*)

(*b*)

Fig. 6. Reduced transverse electric field strength components of TH (*a*) and TE (*b*) modes in $x = 0$ versus the nanoparticle concentration in the active layer p ($n_m = 1.6$, $n_p = 2.5$).

The dependencies of the reduced strengths of the electric field vector of even modes on the volume concentration of nanoparticles in the active layer are displayed in Fig. 6. Here E_x and E_y are divided by corresponding strength amplitudes of the certain mode for the homogeneous waveguide. We can see from the figure that an increase in the concentration of particles p in the active layer leads to a change of the strength of electric field in the central layer of the waveguide, besides the case of basic TE-mode E_{0y} , when E_y practically does not vary. The strength of TE-mode E_{2y} increases with p almost linearly, thus this mode is most promising from the point of view of the observation of nonlinear optical interactions. For TH-modes we observe the competition of two effects: change of the strength of electric field in the active layer with an increase in the concentration of particles and dependence of nonlinear interaction on p . It should be noted that E_{0x} is the nonmonotone function of p , while E_{2x} decreases almost linearly.

From our calculation it can be concluded that the selection of the working mode is essential for the observation of the nonlinear optical effects in waveguides. TE-modes of the symmetrical waveguide are most suitable for the observation of the optical interactions.

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